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ADCT

Apuntes de clase

Apuntes y exámenes ETSIT UPM

Si alguna vez estos apuntes te sirvieron de ayuda, piensa que tus apuntes pueden ayudar a muchas otras personas.

ADCT

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Tema 1: Aplicación Transformada de Laplace al análisis de ctos

1. Definición y propiedades

Sea $f(t)$ de orden exponencial definida para $t \geq 0$

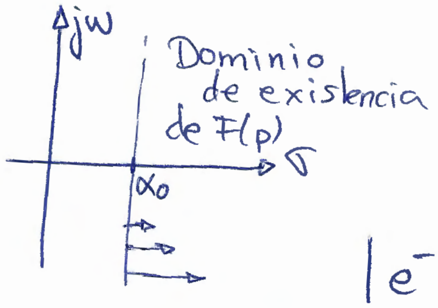
$$|f(t)| \leq M e^{\alpha t}, \forall t > T, \alpha \in \mathbb{R}, M \in \mathbb{R}^+$$

$$F(p) = \int_0^\infty f(t) e^{-pt} dt \quad ; \quad p \in \mathbb{C}$$

$$p = \sigma + jw \; ; \; \sigma, w \in \mathbb{R}$$

$F(p)$ = transformada unilateral de Laplace

→ si $f(t)$ tiene orden exponencial $\Rightarrow$ $F(p)$ converge en $Re[p] \geq \alpha_0$



si $\sigma > \alpha_0 \Rightarrow |F(p)|$ está acotada
La integral converge

$$|e^{-pt}| = e^{-\sigma t}$$

señales siempre causales ($\sigma > \alpha_0$) en esta asignatura!

$$|f(t)| \leq M e^{\alpha_0 t} \; ; \; t > T$$

$$|F(p)| = \left| \int_0^\infty f(t) e^{-pt} dt \right| \leq \left| \int_0^T f(t) e^{-pt} dt \right| + M \int_T^\infty e^{(\alpha_0 - \sigma)t} dt$$

→ Transformada inversa:

$$f(t) = \frac{1}{2\pi j} \int_{\alpha - j\infty}^{\alpha + j\infty} F(p)\, e^{pt}\, dp$$

Válido si $f(t) = 0,\ \forall t < 0$

→ linealidad:

$$k_1 f_1(t) + k_2 f_2(t) \xrightarrow{\mathcal{L}} k_1 F_1(p) + k_2 F_2(p)$$

→ derivación en el tiempo:

$$f'(t) \xrightarrow{\mathcal{L}} pF(p) - f(0)$$

$$f''(t) \xrightarrow{\mathcal{L}} p\left(pF(p) - f(0)\right) - f'(0) = p^2 F(p) - p f(0) - f'(0)$$

$$f^{(n}(t) \xrightarrow{\mathcal{L}} p^n F(p) - \sum_{k=1}^{n} p^{n-k} f^{(k-1}(0)$$

→ integración en el tiempo:

$$f^{(-1}(t) = \int f(t)\, dt \Rightarrow f^{(-1}(t) \xrightarrow{\mathcal{L}} \frac{F(p)}{p} + \frac{f^{(-1}(0)}{p}$$

$$f^{(-n}(t) \xrightarrow{\mathcal{L}} \frac{F(p)}{p^n} - \sum_{k=1}^{n} \frac{f^{(-k}(0)}{p^{n-k+1}}$$

→ desplazamiento temporal:

$$f(t-\tau)\, u(t-\tau) \xrightarrow{\mathcal{L}} F(p)\, e^{-p\tau}$$

válido si $\begin{cases} f(t) = 0, & \forall t < 0 \\ f(t-\tau) = 0, & \forall t < 0 \end{cases}$

→ desplazamiento en el dominio de Laplace:

$$e^{-\alpha t} f(t) \xrightarrow{\mathcal{L}} F(p+\alpha)$$

$$e^{\alpha t} f(t) \xrightarrow{\mathcal{L}} F(p-\alpha)$$

→ Derivación en el dominio de Laplace

$$-t f(t) \xrightarrow{\mathcal{L}} \frac{dF(p)}{dp}$$

$$t f(t) \xrightarrow{\mathcal{L}} -\frac{dF(p)}{dp}$$

$$t^n f(t) \xrightarrow{\mathcal{L}} (-1)^n \frac{d^n F(p)}{dp^n}$$

→ cambio de escala temporal

$$f(at) \xrightarrow{\mathcal{L}} \frac{1}{a} F\left(\frac{p}{a}\right), \quad a>0$$

→ convolución en el dominio del tiempo

$$f_1(t) * f_2(t) \xrightarrow{\mathcal{L}} F_1(p) \cdot F_2(p)$$

2. Transformadas de funciones elementales

$$u(t) \longrightarrow \frac{1}{p}$$

$$k \cdot u(t) \longrightarrow \frac{k}{p}$$

$$t^n u(t) \longrightarrow (-1)^n \frac{d^n\left(\frac{1}{p}\right)}{dp^n} = (-1)^n \frac{d^n p^{-1}}{dp^n} = \frac{n!}{p^{n+1}}$$

$$\sum_{k=0}^{N} a_k t^k u(t) \longrightarrow \sum_{k=0}^{N} a_k \cdot \frac{k!}{p^{k+1}}$$

$$e^{-\alpha t} u(t) \longrightarrow \frac{1}{p+\alpha}$$

$$\text{sen}(\omega_0 t)\, u(t) \longrightarrow \frac{\omega_0}{p^2+\omega_0^2}$$

$$\cos(\omega_0 t)\, u(t) \longrightarrow \frac{p}{p^2+\omega_0^2}$$

$$t^n e^{-\alpha t} u(t) \longrightarrow \frac{n!}{(p+\alpha)^{n+1}}$$

$$e^{-\alpha t} \text{sen}(\omega_0 t)\, u(t) \longrightarrow \frac{\omega_0}{(p+\alpha)^2+\omega_0^2}$$

$$e^{-\alpha t} \cos(\omega_0 t)\, u(t) \longrightarrow \frac{p+\alpha}{(p+\alpha)^2+\omega_0^2}$$

Ej: $t^2 \cdot e^{-\alpha t} \operatorname{sen}(\omega_0 t)\, u(t), \quad n \in \mathbb{N}, \quad \alpha, \omega_0 \in \mathbb{R}$

$$t^2 e^{-\alpha t}\left(\frac{1}{2j}\left(e^{j\omega_0 t} - e^{-j\omega_0 t}\right)\right) u(t) =$$

$$= \frac{1}{2j} t^2 \left(e^{-(\alpha - j\omega_0)t} - e^{-(\alpha + j\omega_0)t}\right) u(t) \longrightarrow$$

$$\left(t^2 \longrightarrow \frac{2}{p^3}\right)$$

$$\longrightarrow \frac{1}{j}\left[\frac{1}{(p+\alpha-j\omega_0)^3} - \frac{1}{(p+\alpha+j\omega_0)^3}\right]$$

3. Ecuaciones integro-diferenciales

↳ Resolución en el dominio del tiempo ☹

↳ Resolución utilizando Laplace ☺

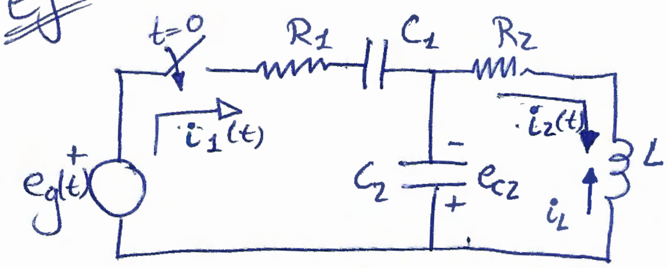
$$A\frac{dy(t)}{dt} + B\,y(t) + C\int y(t)\,dt = f(t)$$

$$\downarrow \mathcal{L} \qquad \downarrow \mathcal{L} \qquad \downarrow \mathcal{L} \qquad \searrow \mathcal{L}$$

$$A\left(p\,Y(p) - y(0)\right) + B\,Y(p) + C\left(\frac{1}{p}Y(p) + \frac{y^{(-1)}(0)}{p}\right) = F(p)$$

↳ hallaremos $Y(p) \xrightarrow{\mathcal{L}^{-1}} y(t)$

Ej:



$e_{c2}(0) = E_c$

$i_L(0) = I_L$

$e_{c1}(0) = 0$

por mallas, para $t \geq 0$:

$$R_1 i_1(t) + \frac{1}{C_1}\int_0^t i_1(t)\,dt + \cancel{e_{c1}(0)} + \frac{1}{C_2}\int\left(i_1(t) - i_2(t)\right)dt - e_{c2}(0) = e_g(t)$$

$\downarrow \mathcal{L}$

$$R_1 I_1(p) + \frac{1}{C_1 p} I_1(p) + \frac{1}{C_2 \cdot p}\left(I_1(p) - I_2(p)\right) - \frac{e_{c2}(0)}{p} = E_g(p)$$

$$R_2\, i_2(t) + L\,\frac{di_2(t)}{dt} + e_{c2}(t) = 0$$

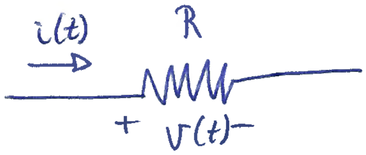
$$R_2\, i_2(t) + L\,\frac{di_2(t)}{dt} + \frac{1}{C_2}\int_0^t (i_2(t) - i_1(t))\,dt + e_{c2}(0) = 0$$

$$R_2\, I_2(p) + L\left(p\,I_2(p) - i_2(0)\right) + \frac{1}{C_2 p}\left(I_2(p) - I_1(p)\right) + \frac{e_{c2}(0)}{p} = 0$$

$$R_2\, I_2(p) + L\left(p\,I_2(p) + I_L\right) + \frac{1}{C_2 p}\left(I_2(p) - I_1(p)\right) + \frac{E_c}{p} = 0$$

4. Relación I-V de R, L, C en Laplace

Resistencia:

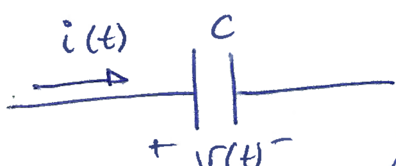


$$v(t) = i(t)\cdot R$$

$$V(p) = I(p)\cdot R$$

$$Z(p) = \frac{V(p)}{I(p)} = R$$

Capacidad:



$$i_c(t) = C\cdot\frac{dV_c(t)}{dt}$$

$$v(t) = \frac{1}{C}\int i(t)\,dt + v(0)$$

$$V(p) = \frac{1}{C\cdot p}\,I(p) + \frac{v(0)}{p}$$

$$Z(p) = \frac{V(p)}{I(p)} = \frac{1}{C\cdot p}$$

Bobina:

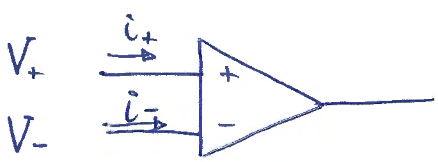
$$v(t) = L\,\frac{di(t)}{dt}$$

$$i(t) = \frac{1}{L}\int v(t)\,dt + i(0)$$

$$I(p) = \frac{1}{L\cdot p}\,V(p) + \frac{i(0)}{p}$$

$$Z(p) = \frac{V(p)}{I(p)} = L\cdot p$$

Amp. Op. ideal:



$$V_+(t) = V_-(t) \qquad i_+(t) = i_-(t) = 0$$

$$V_+(p) = V_-(p) \qquad I_+(p) = I_-(p) = 0$$

5. Kirchoff

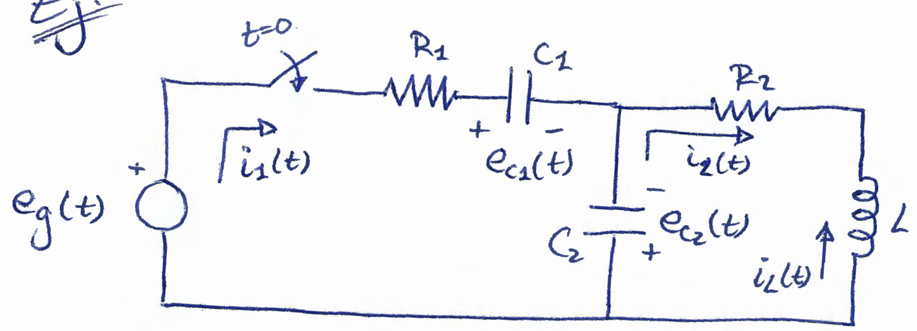
nudos: $\sum_k i_k(t) = 0 \longleftrightarrow \sum_k I_k(p) = 0$

mallas: $\sum_k v_k(t) = 0 \longleftrightarrow \sum_k V_k(p) = 0$

6. Análisis

→ condiciones iniciales
→ circuito en el dominio Laplace
→ resolución en Laplace
→ resolución algebraica
→ solución temporal

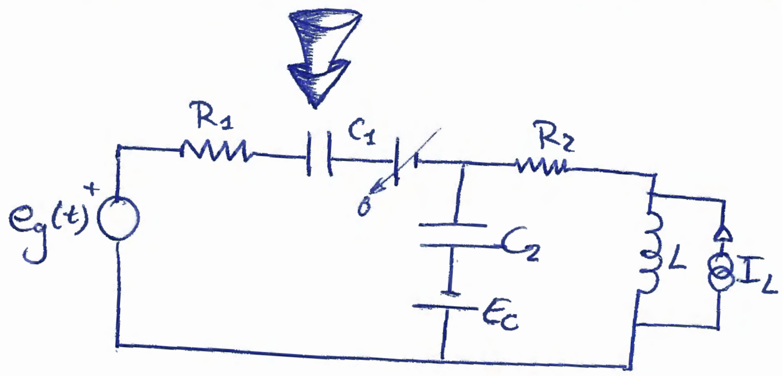
Ej:

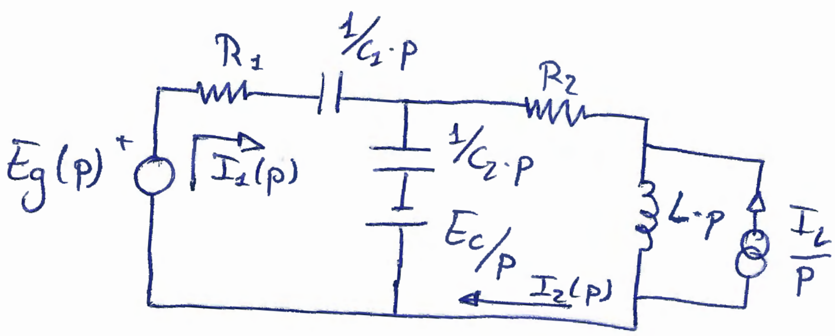


$e_{c1}(0) = 0$

$e_{c2}(0) = E_c$

$i_L(0) = I_L$





$$R_1 I_1(p) + \frac{I_1(p)}{C_1 \cdot p} + \frac{I_1(p) - I_2(p)}{C_2 \cdot p} - \frac{E_c}{p} = E_g(p)$$

$$R_2 I_2(p) + L \cdot p \left(I_2(p) + \frac{I_L}{p} \right) + \frac{E_c}{p} + \frac{I_2(p) - I_1(p)}{C_2 \cdot p} = 0$$

7. Teoremas límites

Teorema del valor inicial:

$$\text{si } f(t) \longrightarrow F(p) \Rightarrow \lim_{t \to 0^+} f(t) = \lim_{p \to \infty} p \cdot F(p)$$

Teorema del valor final:

$$\text{si } f(t) \longrightarrow F(p) \Rightarrow \lim_{t \to \infty} f(t) = \lim_{p \to 0} p \cdot F(p)$$

necesario que $pF(p)$ sea analítica en todo σ^+ es decir, que contenga al eje $j\omega$

8. Transformada inversa

→ $$f(t) = \frac{1}{2\pi j} \int_{\alpha - j\infty}^{\alpha + j\infty} F(p) e^{pt} \, dp$$

→ Cálculo de residuos:

$$\lim_{R \to} \frac{1}{2\pi j} \oint_{C_1 + C_2} F(p) e^{pt} \, dp = \sum \text{res} \{ F(p) \cdot e^{pt} \}$$

$$f(t) = \frac{1}{2\pi j} \int_{\alpha - j\infty}^{\alpha + j\infty} F(p) e^{pt} \, dp = \sum \text{res} \{ F(p) e^{pt} \}$$

residuo de $H(p)$ de un polo de orden N:

$$\text{res}\{H(p)\} = \lim_{p \to p_0} \left(\frac{1}{(N-1)!} \frac{d^{N-1}}{dp^{N-1}} \left((p - p_0)^N H(p) \right) \right)$$

Ej:

$$F(p) = \frac{6p^5 + 25p^4 + 62p^3 + 100p^2 + 92p + 40}{(p^2+4)(p^2+2p+2)(p+1)^2}$$

Cálculo de polos:

$p^2+4 = 0 \iff p_1 = 2j$ (orden 1)
$p_2 = -2j$ (orden 1)

$(p+1)^2 = 0 \iff p_5 = -1$ (orden 2)

$p^2+2p+2 = 0 \iff p_3 = -1+j$
$p_4 = -1-j$

$$k_1(t) = \lim_{p \to 2j} \left[(p-2j) F(p) e^{pt} \right] = (1-j) e^{2jt} = \sqrt{2} e^{-j\frac{\pi}{4}} e^{2jt}$$

$$k_2(t) = \lim_{p \to -2j} \left[(p+2j) F(p) e^{pt} \right] = (1+j) e^{-2jt} = k_1^*(t)$$
$$= \sqrt{2} e^{j\frac{\pi}{4}} e^{-2jt}$$

- $k_1(t) + k_2(t) = 2\sqrt{2} \cos\left(2t - \frac{\pi}{4}\right)$

$$k_3(t) = \lim_{p \to (-1+j)} \left[(p+1-j) F(p) e^{pt} \right] = e^{-(1+j)t} = e^{-t} e^{-jt}$$

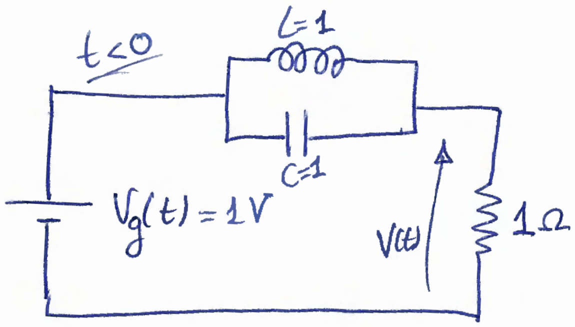
$$k_4(t) = e^{-(1-j)t} = e^{-t} e^{jt}$$

- $k_3(t) + k_4(t) = e^{-t} 2 \cos(t)$
- $k_5(t) = \lim_{p \to -1} \frac{d}{dp} \left[(p+1)^2 F(p) e^{pt} \right] = (2+t) e^{-t}$

$$f(t) = \Big[\underbrace{2\sqrt{2} \cos\left(2t - \frac{\pi}{4}\right)}_{\substack{K_1(t) + K_2(t) \\ Res(p_1) + Res(p_2)}} + \underbrace{2 e^{-t} \cos(t)}_{\substack{K_3(t) + K_4(t) \\ Res(p_3) + Res(p_4)}} + \underbrace{(2+t) e^{-t}}_{\substack{k_5(t) \\ Res(p_5)}} \Big] \cdot u(t)$$

Ej 2:

$t<0$



En $t=0$ $V_g(t)$ invierte polaridad

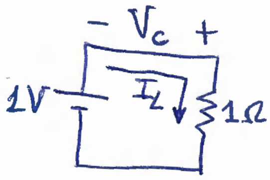
a) $\mathcal{L}$ de $v(t)$?

b) th. valor final $v(t)$ razonar físicamente

c) dibujar $v(t)$

d) incluir  en serie con 

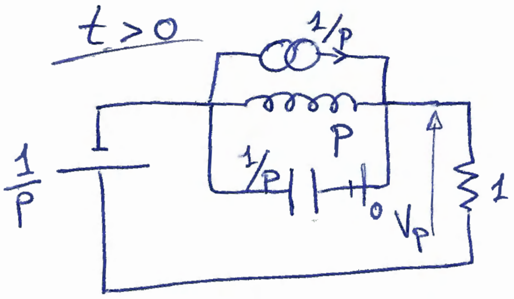
a) $t<0$

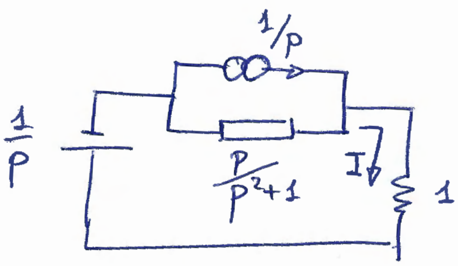


$V_c(t=0)=0$

$i_L(t=0)=\frac{1V}{1\Omega}=1A$

$t>0$





$I(p)=I$

$$\frac{1}{p}+\left(\frac{p}{p^2+1}\right)\left(I-\frac{1}{p}\right)+I=0 \Rightarrow I=\frac{-p^2+p-1}{p(p^2+p+1)}$$

$$\mathcal{L}(v(t))=V(p)=\frac{-p^2+p-1}{p(p^2+p+1)}=I(p)\cdot 1\Omega$$

b) $\lim_{t\to\infty} v(t)=\lim_{p\to 0} p\cdot V(p) \Leftrightarrow p\,V(p)$ tiene polos en σ^-

$\hookrightarrow p^2+p+1=0 \Rightarrow p=-\frac{1}{2}\pm j\frac{\sqrt{3}}{2}$

$$\lim_{p\to 0} p\,V(p)=\lim_{p\to 0}\frac{-p^2+p-1}{p^2+p+1}=-1$$

c) $\lim_{t \to 0} v(t) = \lim_{p \to \infty} p \cdot V(p) = -1$

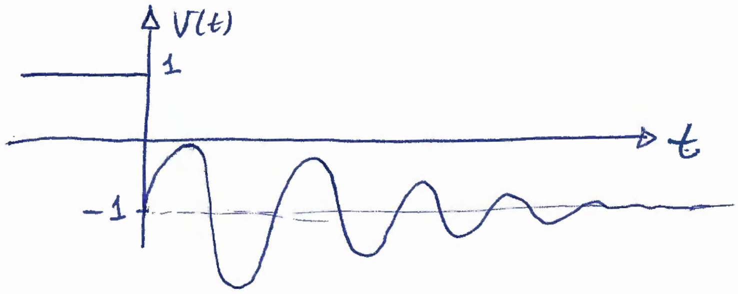
Polos:

$p_1 = 0$

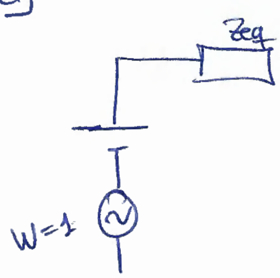
$p_2 = -\frac{1}{2} + j\frac{\sqrt{3}}{2}$

$p_3 = \frac{-1}{2} - j\frac{\sqrt{3}}{2}$

$$v(t) = k_1 + k_2\, e^{-t/2} \cdot \cos\left(\frac{\sqrt{3}}{2} t + \varphi\right)$$

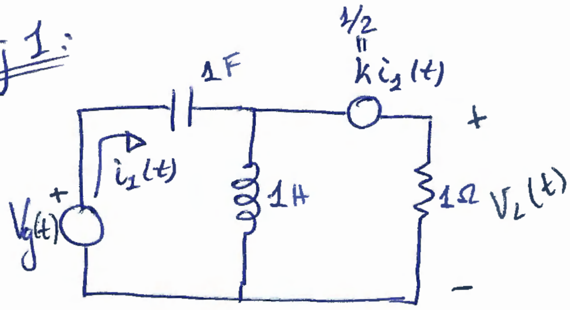


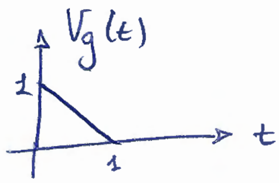
d)



$$Z_{eq} = \frac{jw \cdot \frac{1}{jw}}{jw + \frac{1}{jw}} = \frac{jw}{1-w^2} = \{w=1\} = \infty$$

Ej 1:





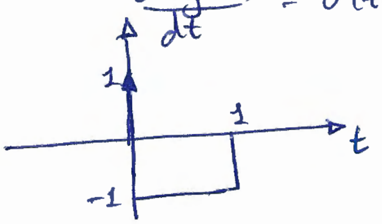
a) $V_g(p)$

b) $V_L(p)$

c) $v_L(t)$

a) $$\frac{dV_g(t)}{dt} = \delta(t) - u(t) + u(t-1) \longleftrightarrow 1 - \frac{1}{p} + \frac{1}{p} e^{-p}$$

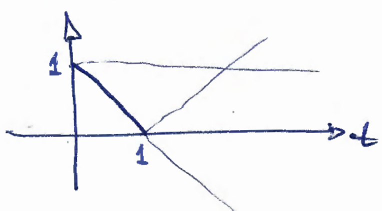
Integrando



$$V_g(p) = \frac{1}{p} - \frac{1}{p^2}\left(e^{-p} - 1\right)$$

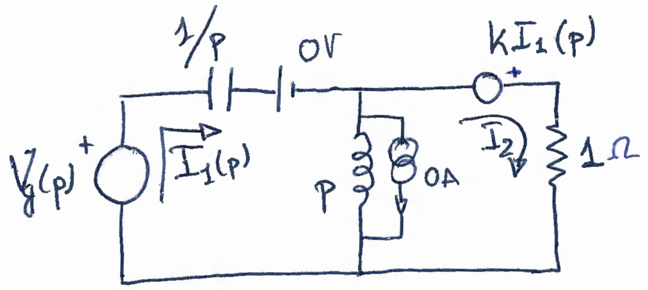
Another way:

$$V_g(t) = u(t) - t\,u(t) + (t-1)\,u(t-1)$$



$$V_g(p) = \frac{1}{p} - \frac{1}{p^2} + \frac{1}{p^2} e^{-p} = \frac{1}{p} + \frac{1}{p^2}\left(e^{-p} - 1\right)$$

b)



$$\left.\begin{array}{l} V_g = I_1\left(\frac{1}{p}+p\right) - I_2\, p \\ kI_1 = (1+p) I_2 - p I_1 \end{array}\right\}$$

$$I_2 = V_g \cdot \frac{p(p+k)}{p^2(1-k)+p+1} \quad ; k = \frac{1}{2}$$

$$V_L = I_2 \cdot 1\Omega = I_2$$

$$V_L = \left[\frac{1}{p} + \frac{1}{p^2}\left(e^{-p}-1\right)\right] \cdot \frac{p(p+k)}{p^2(1-k)+p+1} = \underbrace{\frac{2p+1}{p^2+2p+2}}_{V_1(p)} +$$

$$-\underbrace{\frac{2p+1}{p(p^2+2p+2)}}_{V_2(p)} + \underbrace{\frac{(2p+1)\,e^{-p}}{p(p^2+2p+2)}}_{V_3(p)}$$

$$V_L = V_1(p) - V_2(p) + V_3(p) \longrightarrow V_1(t) - V_2(t) + V_2(t-1)$$

$$V_1(p) = \frac{2p+1}{p^2+2p+2} \longrightarrow \text{polos en: } p_{1,2} = -1 \pm j$$

$$p_1 = -1+j \Rightarrow k_1(t) = \lim_{p\to -1+j} (p+1-j)\, V_p \cdot e^{pt} =$$

$$= \lim_{p\to -1+j} \frac{2p+1}{p+1+j} \cdot e^{pt} = (1+j/2)\, e^{-t} e^{jt}$$

$$= \sqrt{\frac{5}{4}} \cdot e^{-t} \cdot e^{j(t-\mathrm{arctg}(1/2))} = k_2(t)$$

$$V_1(t) = 2\sqrt{\frac{5}{4}}\; e^{-t} \cos\left(t - \mathrm{arctg}(1/2)\right) \cdot u(t)$$

$k_1(t)$ y $k_2(t)$

$$V_2(p) = \frac{2p+1}{p(p^2+2p+2)} \longrightarrow \text{polos en: } p_1 = 0 \;,\; p_{2,3} = -1 \pm j$$

$$k_1(t) = \lim_{p\to 0} \frac{2p+1}{p^2+2p+2} \cdot e^{pt} = \frac{1}{2}$$

$$k_2(t) = \lim_{p\to -1+j} \frac{2p+1}{p(p+1+j)}\, e^{pt} = \frac{-1-3j}{4}\, e^{-t} e^{jt}$$

$$= \sqrt{\frac{10}{16}}\, e^{-t} e^{j(t+\mathrm{arctg}(3))}$$

$$V_2(t) = \left[\frac{1}{2} + 2\sqrt{\frac{10}{16}}\, e^{-t} \cos(t + \operatorname{arctg}(3)) \right] u(t)$$

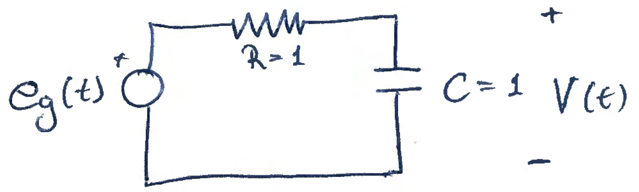
Ej 3:

$$e_g(t) = |\operatorname{sen}(\pi t)| \cdot u(t)$$

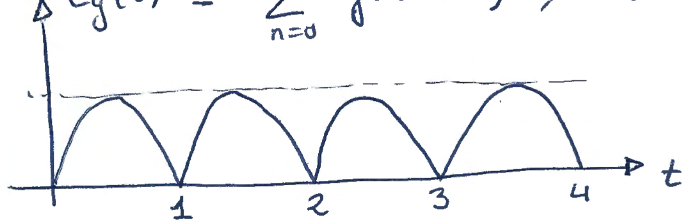
a) $E_g(p)$

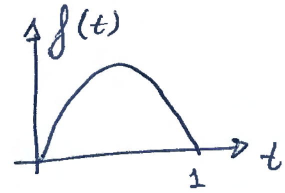
b) $V(p)$, diagrama polos, ceros
th. valor inicial y final

c) $V(t)$, regimen permanente?
razonar físicamente



$$e_g(t) = \sum_{n=0}^{\infty} f(t-nT) \quad ; \quad T=1$$





$$E_g(p) = \sum_{n=0}^{\infty} F(p)\, e^{-n \cdot T \cdot p} = F(p) \sum_{n=0}^{\infty} e^{-nTp} \quad ; \quad T=1$$

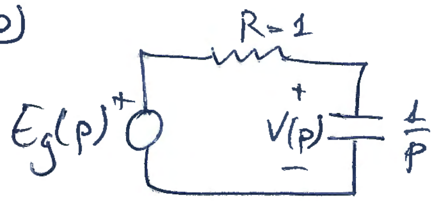
$$= F(p) \cdot \frac{1-0}{1-e^{-p}}$$

También: $f(t) = \operatorname{sen}(\pi t)\, u(t) + \operatorname{sen}(\pi(t-1))\, u(t-1)$

$$F(p) = \frac{\pi}{p^2+\pi^2} + \frac{\pi}{p^2+\pi^2} e^{-p} = (1+e^{-p}) \frac{\pi}{p^2+\pi^2}$$

$$E_g(p) = \frac{1+e^{-p}}{1-e^{-p}} \cdot \frac{\pi}{p^2+\pi^2}$$

b)

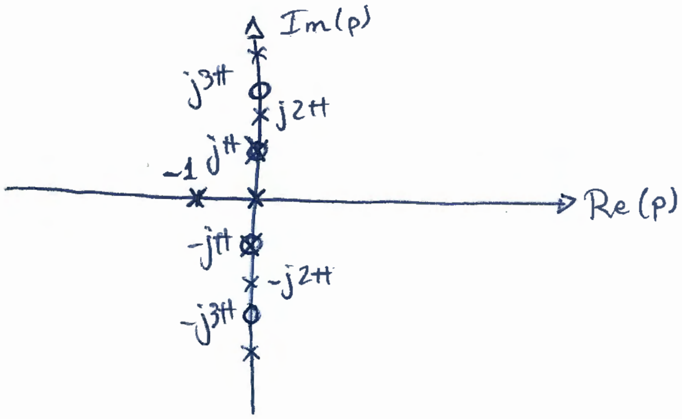


divisor de tensión

$$V(p) = \frac{1/p}{1+1/p} \cdot E_g(p) = \frac{E_g}{p+1} =$$

$$= \frac{1+e^{-p}}{1-e^{-p}} \cdot \frac{\pi}{(p^2+\pi^2)(p+1)}$$

ceros: $1+e^{-p}=0 \rightarrow p=j(\pi+2n\pi)$ $\quad \infty$ ceros



polos: $p^2+\pi^2=0 \rightarrow p=\pm j\pi$

$p+1=0 \rightarrow p=-1$

$1-e^{-p}=0 \longrightarrow p=0 \;,\; p=j2\pi n$

$$\lim_{t\to 0} V(t) = \lim_{p\to\infty} p\cdot V(p) = \lim_{p\to\infty} \frac{p\cdot\pi(1+e^{-p})}{(p+1)(p^2+\pi^2)(1-e^{-p})} = 0$$

No se puede aplicar el th. valor final.

c) $p=-1$ $\quad K_{-1}(t) = \lim_{p\to -1} (p+1)\, V(p)\, e^{pt} =$

$$= \lim_{p\to -1} \frac{\pi(1+e^{-p})}{(p^2+\pi^2)(1-e^{-p})} e^{pt} = \frac{\pi(1+e)}{(1+\pi^2)(1-e)} e^{-t}$$

$p=0$ $\quad \lim_{p\to 0} p\cdot V(p)\, e^{pt} = \lim_{p\to 0} \frac{\cancel{p}\,\pi(1+e^{-p})}{(p+1)(p^2+\pi^2)\cancel{(1-e^{-p})}} e^{pt}$

$$= \lim_{p\to 0} \frac{\pi(1+e^{-p})}{(p+1)(p^2+\pi^2)} e^{pt} = \frac{2}{\pi}$$

$p_n = j2\pi n \;,\; n=\pm 1, \pm 2, \pm 3 \ldots$

$$K_n(t) = \lim_{p\to j2\pi n} \frac{\cancel{(p-j2\pi n)}\,\pi(1+e^{-p})}{(p+1)(p^2+\pi^2)\cancel{(1-e^{-p})}} e^{pt} =$$

$$= \frac{2\pi}{(1+j2\pi n)(\pi^2-4\pi^2 n^2)} e^{j2\pi n t}$$

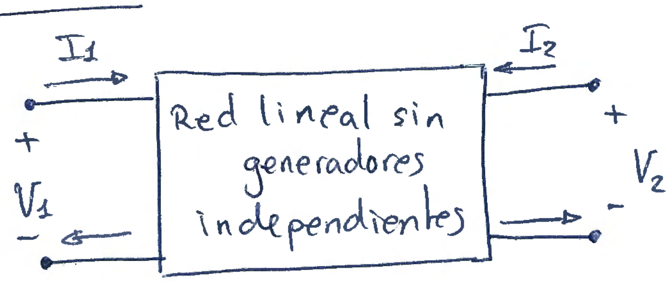
$$V(t) = \frac{\pi(1+e)}{(1+\pi^2)(1-e)}\, e^{-t} u(t) + \frac{2}{\pi} u(t) + \sum_{n=1}^{\infty} \frac{4\cos(2\pi n t + \varphi_n)}{\pi(1-4n^2)\sqrt{1+4\pi^2 n^2}}\, u(t)$$

$$\varphi_n = -\operatorname{arctg}(2\pi n)$$

Tema 2: Cuadripolos y filtros

1. Cuadripolos

Definición:



Familias de parámetros:

↳ Familia [Z] (Matriz de impedancias)

$$\left.\begin{aligned} V_1 &= z_{11}I_1 + z_{12}I_2 \\ V_2 &= z_{21}I_1 + z_{22}I_2 \end{aligned}\right\} \quad \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = [Z]\begin{pmatrix} I_1 \\ I_2 \end{pmatrix}$$

$$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} \qquad z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0}$$

$$z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} \qquad z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$

↳ Familia [Y] (Matriz de admitancias)

$$\left.\begin{aligned} I_1 &= y_{11}V_1 + y_{12}V_2 \\ I_2 &= y_{21}V_1 + y_{22}V_2 \end{aligned}\right\} \quad \begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = \begin{pmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = [Y]\begin{pmatrix} V_1 \\ V_2 \end{pmatrix}$$

$$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} \qquad y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0}$$

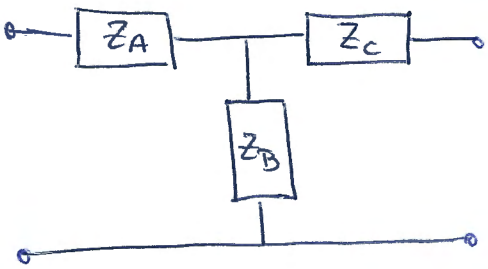
$$y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} \qquad y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$$

Parámetros [h]:

$$\left.\begin{aligned} V_1 &= h_{11} I_1 + h_{12} V_2 \\ I_2 &= h_{21} I_1 + h_{22} V_2 \end{aligned}\right\} \quad \begin{pmatrix} V_1 \\ I_2 \end{pmatrix} = [h] \begin{pmatrix} I_1 \\ V_2 \end{pmatrix}$$

Cuadripolos en T y en П:

Cuadripolo en T:

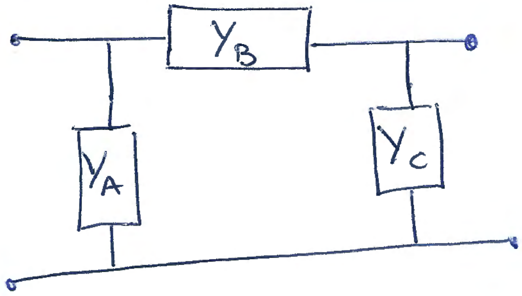


$$z_{11} = Z_A + Z_B$$

$$z_{12} = z_{21} = Z_B$$

$$z_{22} = Z_B + Z_C$$

Cuadripolo en П:



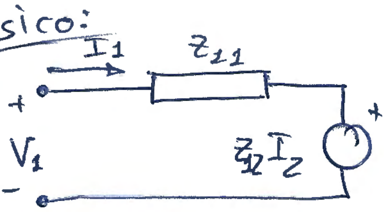
$$y_{11} = Y_A + Y_B$$

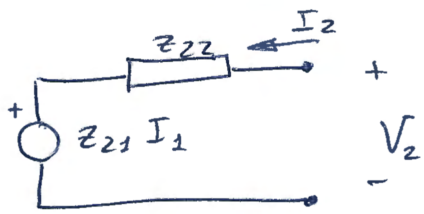
$$y_{12} = y_{21} = -Y_B$$

$$y_{22} = Y_B + Y_C$$

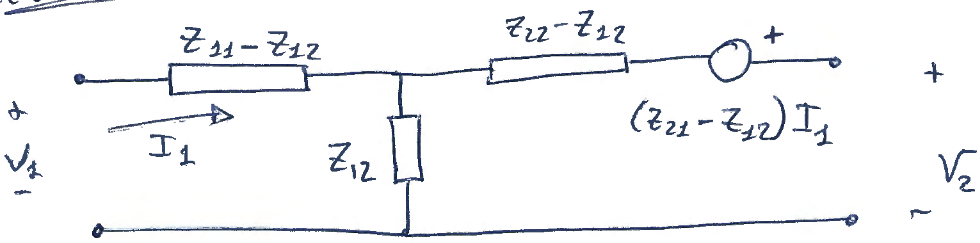
Circuitos equivalentes:

Básico:



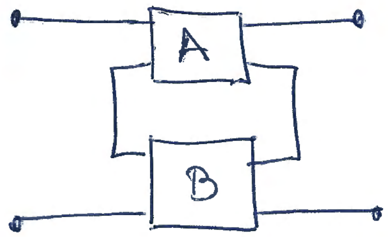


Célula en T:



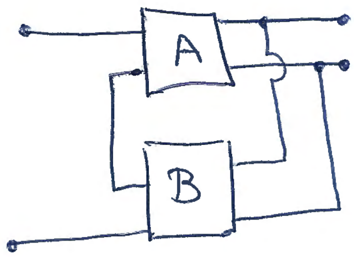
Asociación de cuadripolos:

* serie-serie ~ serie



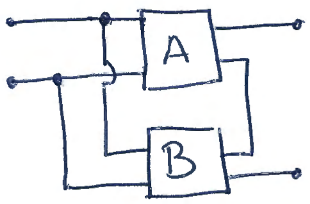
$[Z_T] = [Z_A] + [Z_B]$

* serie-paralelo



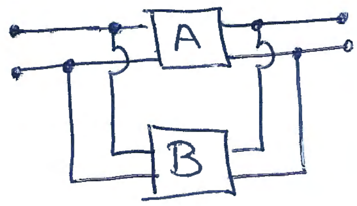
$[H_T] = [H_A] + [H_B]$

* paralelo-serie



$[G_T] = [G_A] + [G_B]$

* paralelo-paralelo ~ paralelo



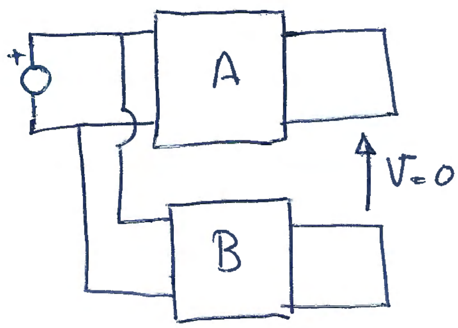
$[Y_T] = [Y_A] + [Y_B]$

- Condiciones de Brune:

$$\begin{cases} [z] = [z_A] + [z_B] \\ [h] = [h_A] + [h_B] \\ [g] = [g_A] + [g_B] \\ [y] = [y_A] + [y_B] \end{cases}$$

- Cálculo de las condiciones de Brune:

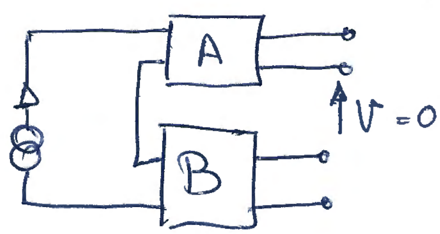
opc1 paralelo



cortocircuitamos salidas

si $V = 0 \Rightarrow$ se cumple Brune

opc2 serie

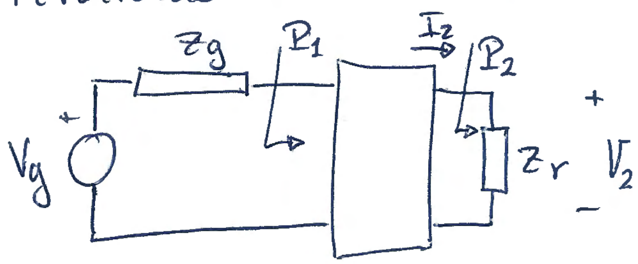


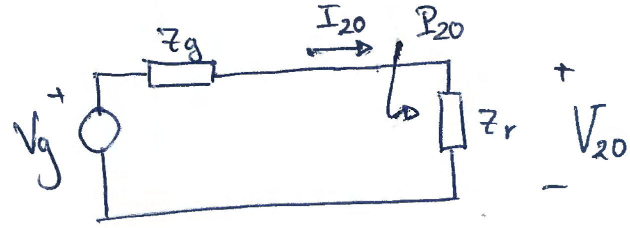
dejamos las salidas abiertas

si $V = 0 \Rightarrow$ se cumple Brune

Relaciones de potencia:

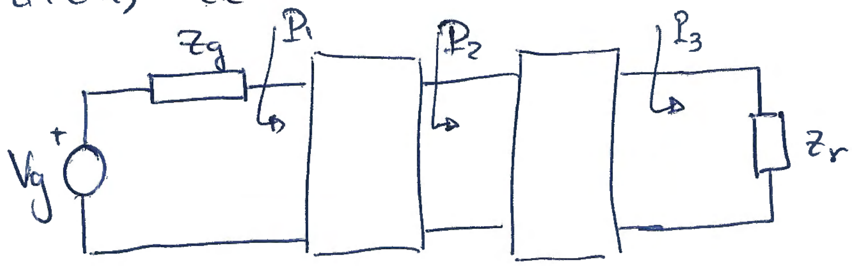
Pérdidas de inserción:





$$\left(\frac{P_{20}}{P_2}\right)_{dB} = 10 \cdot \log_{10}\left(\frac{P_{20}}{P_2}\right) = 20 \log_{10}\left(\frac{V_{20}}{V_2}\right) = 20 \log_{10}\left(\frac{I_{20}}{I_2}\right)$$

Pérdidas de transmisión:

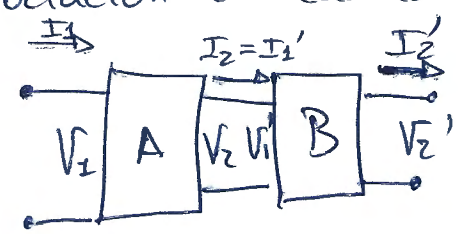


$$\left(\frac{P_1}{P_3}\right)_{dB} = \left(\frac{P_1}{P_2} \cdot \frac{P_2}{P_3}\right)_{dB} = 10 \cdot \log_{10}\left(\frac{P_1}{P_2} \cdot \frac{P_2}{P_3}\right)$$

Parámetros [F]:

toman como variables independientes la tensión y corriente en la puerta 2. La corriente I_2 se toma saliente!!

Asociación en cascada:



$$\begin{pmatrix} V_1 \\ I_1 \end{pmatrix} = [F_A] \begin{pmatrix} V_2 \\ I_2 \end{pmatrix} = [F_A]\,[F_B] \begin{pmatrix} V_2' \\ I_2' \end{pmatrix}$$

Tipos de cuadripolos:

→ Pasivos y recíprocos: la relación respuesta/excitación es invariante frente a un cambio de posición

$$z_{12} = z_{21} \qquad y_{12} = y_{21}$$

$$h_{12} = -h_{21} \qquad g_{12} = -g_{21}$$

$$|F| = AD - BC = 1$$

→ Pasivos y simétricos: la entrada y salida son intercambiables

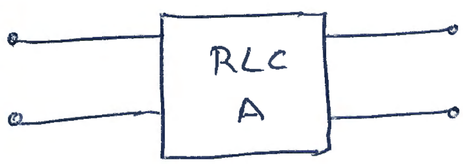
$$z_{11} = z_{22} \qquad y_{11} = y_{22}$$

$$h_{11} \cdot h_{22} - h_{12} h_{21} = 1$$

$$g_{11} g_{22} - g_{12} g_{21} = 1$$

$$A = D$$

Ej:



Medida 1: Puerta 1 en circuito abierto

$$v_2(t) = \cos(t)$$

$$i_2(t) = 0'5 \cdot \text{sen}(t)$$

Medida 2: Puerta 1 en cortocircuito

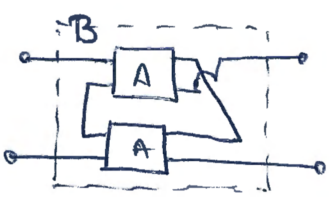
$$v_2(t) = \cos(t)$$

$$i_2(t) = -2\,\text{sen}(t)$$

$$i_1(t) = -\text{sen}(t)$$

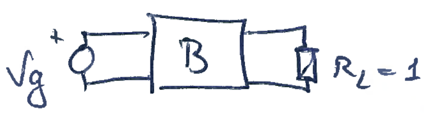
a) Z_A? $\omega = 1$

b)



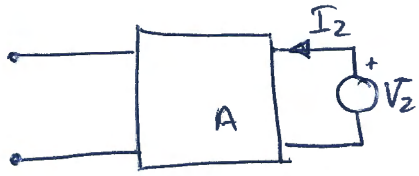
Z?

c)



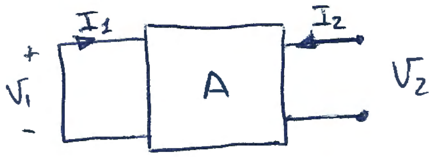
$V_g(t) = \cos(t)$

a) medida 1:



$$\frac{V_2}{I_2} = Z_{22} = \frac{1 = V_2}{-0'5j = I_2} = 2j$$

medida 2:



$$\left.\begin{array}{l} V_2 = 1 \\ I_2 = 2j \\ I_1 = j \\ V_1 = 0 \end{array}\right| \quad \begin{array}{l} 0 = Z_{11} \cdot j + Z_{12} \cdot 2j \\ 1 = Z_{21} \cdot j + Z_{22} \cdot 2j \\ 1 = Z_{21} j + (-4) \end{array}$$

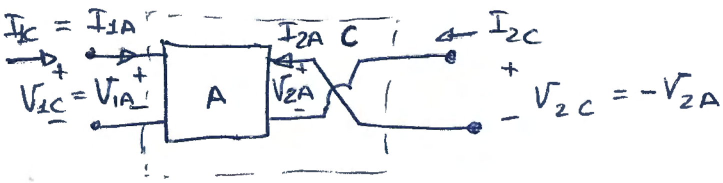
$$\hookrightarrow Z_{21} = \frac{5}{j} = -5j$$

$$Z_{12} = -5j$$

$$Z_{11} = 10j$$

$$Z_A = \begin{pmatrix} 10j & -5j \\ -5j & 2j \end{pmatrix}$$

b)



$$\left.\begin{array}{l} I_{1C} = I_{1A} \\ V_{1C} = V_{1A} \\ I_{2C} = -I_{2A} \\ V_{2C} = -V_{2A} \end{array}\right\}$$

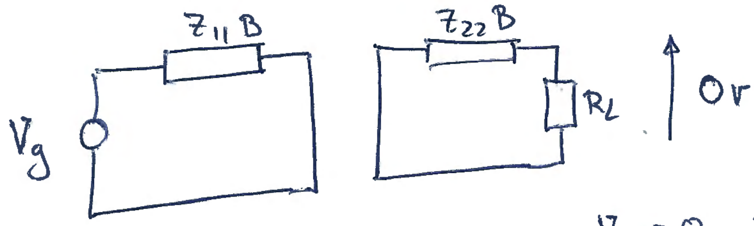
$$V_{1A} = Z_{11A} \cdot I_{1A} + Z_{12A} \cdot I_{2A}$$

$$V_{2A} = Z_{21A} \cdot I_{1A} + Z_{12A} \cdot I_{2A}$$

$$\left.\begin{array}{l} V_{1C} = V_{1A} = Z_{11A} I_{1C} - Z_{12A} \cdot I_{2C} \\ V_{2C} = -V_{2A} = -Z_{21A} I_{1C} + Z_{22A} I_{2C} \end{array}\right\} Z_C = \begin{pmatrix} Z_{11A} & -Z_{12A} \\ -Z_{21A} & Z_{22A} \end{pmatrix}$$

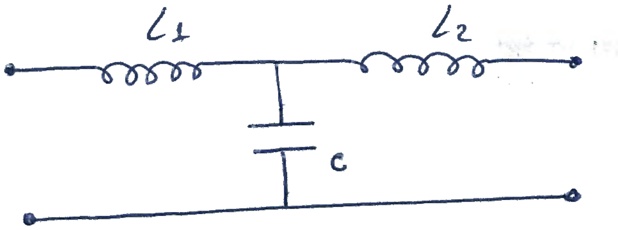
$$\begin{pmatrix} 20j & 0 \\ 0 & 4j \end{pmatrix} = Z_B = \begin{pmatrix} 2Z_{11A} & 0 \\ 0 & 2Z_{22A} \end{pmatrix}$$

c)



$V_2 = 0 \Rightarrow v_2(t) = 0$

Ej:



a) $Y_{21} = Y_{12}$?? (reciproco)

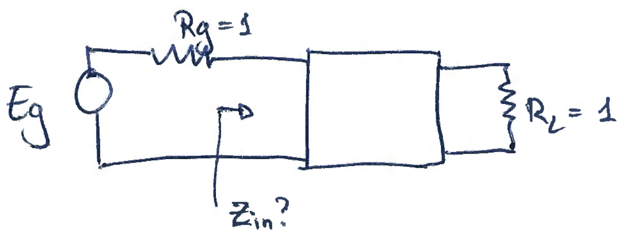
condición para que sea simétrico ($Y_{11} = Y_{22}$)?

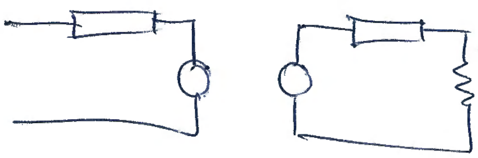
sí reciproco por estar formado sólo por elementos pasivos

$Z_{11} = L_1 \cdot s + \frac{1}{C \cdot s}$, $Z_{22} = L_2 \cdot s + \frac{1}{C \cdot s}$

$Z_{12} = Z_{21} = \frac{1}{C \cdot s}$

b)



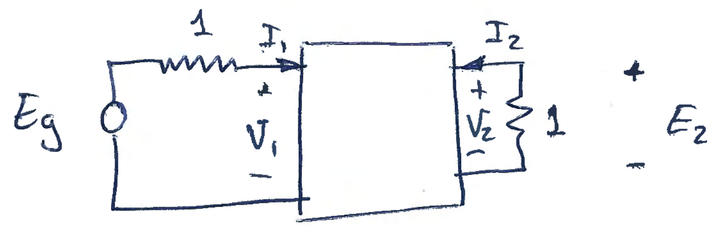


$$Z_{in} = Z_{11} - \frac{Z_{21} \cdot Z_{12}}{R_L + Z_{22}}$$

$$Z_{in} = L_1 \cdot s + \frac{1}{C \cdot s} - \frac{\frac{1}{C \cdot s} \cdot \frac{1}{C \cdot s}}{R_L + L_2 s + 1/Cs} =$$

$$= L_1 s + \frac{R_L + L_2 s}{C s \left(R_L + L \cdot s + \frac{1}{C \cdot s}\right)}$$

c)

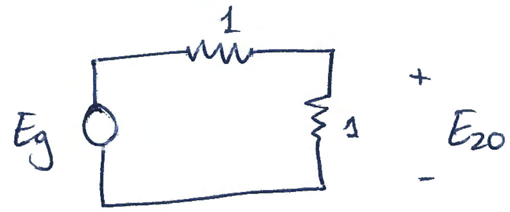


$L_1 = L_2 = 1$

$C = 2$

$R_g = R_L = 1$

$$\left(\frac{P_{20}}{P_2}\right)_{dB} = 10\log_{10}\left(\frac{P_{20}}{P_2}\right) = 20\log\frac{|E_{20}|}{|E_2|}$$



$$\Rightarrow E_{20} = \frac{E_g}{2}$$

$$V_1 = \left(s + \frac{1}{2\cdot s}\right) I_1 + \frac{1}{2\cdot s} I_2$$

$$E_2 = \frac{1}{2\cdot s} I_1 + \left(s + \frac{1}{2\cdot s}\right) I_2$$

$$E_2 = -I_2$$

$$E_g = I_1 + V_1$$

$$E_g = \left(s + \frac{1}{2s} + 1\right) I_1 + \frac{1}{2s} I_2$$

$$E_2 = \frac{1}{2s} I_1 - \left(s + \frac{1}{2s}\right) E_2 \Rightarrow E_2\left(s + \frac{1}{2s} + 1\right) = \frac{1}{2s} I_1$$

$$E_g = \left(s + \frac{1}{2s} + 1\right)(2s^2 + 2s + 1) E_2 - \frac{E_2}{2s}$$

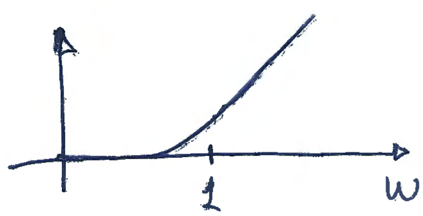
$$E_g = \left((2s^2 + 2s + 1)^2 - 1\right) E_2 \; ; \quad \frac{E_2}{E_g} = \frac{2s}{(2s^2+2s+1)^2 - 1}$$

$$\frac{E_{20}}{E_2} = \frac{E_{20}/E_g}{E_2/E_g} = \frac{1}{4s}\left(4s^4 + 8s^3 + 8s^2 + 4s + 1 - 1\right) =$$

$$= s^3 + 2s^2 + 2s + 1 = (s+1)(s^2+s+1)$$

$$\left.\frac{E_{20}}{E_2}\right|_{s=j\omega} = (j\omega + 1)(-\omega^2 + j\omega + 1)$$

$$\left|\frac{E_{20}}{E_2}\right|^2 = (\omega^2+1)\left((1-\omega^2)^2 + \omega^2\right) = 1 + \omega^6$$



2. Respuesta temporal y frecuencial

Un sistema será estable si sus polos se encuentran en $Re[s] = \sigma < 0$, siendo así su solución causal la que contenga al eje jw.

si el sistema es estable: $H(jw) = H(s)|_{s=jw}$

respuesta en amplitud: $|H(jw)|$

respuesta en fase: $\arg(H(jw))$

Sea la respuesta en frecuencia:

$$H(s) = k\frac{(s+z_1)(s+z_2)\cdots}{s(s+p_1)(s+p_2)\cdots} = k\frac{(s-(-s_{c1}))(s-(-s_{c2}))\cdots}{s(s-(-s_{p1}))(s-(-s_{p2}))\cdots}$$

$$\xrightarrow{s=jw} H(jw) = k\frac{(jw+z_1)(jw+z_2)\cdots}{jw(jw+p_1)(jw+p_2)\cdots}$$

$$\rightarrow H(jw) = k'\frac{(1+jw/z_1)(1+jw/z_2)\cdots}{jw(1+jw/p_1)(1+jw/p_2)\cdots}$$

la respuesta en frecuencia queda como:

$$H(jw) = k'(1+jw/z_1)(1+jw/z_2)\frac{1}{jw}\cdot\frac{1}{(1+jw/p_1)}\cdot\frac{1}{1+jw/p_2}\cdots$$

El diagrama de BODE representa amplitud y fase agregando las contribuciones de cada término

Amplitud (en dB):

$$w_1 \in \mathbb{R}\begin{cases} 20\cdot\log_{10}(|H(jw)|) = 20\cdot\log_{10}(|K'|) + 20\log_{10}(|1+jw/z_1|) + \cdots \\ 20\log_{10}(|1+jw/w_1|) = 20\cdot\log_{10}\left(\sqrt{1+\frac{jw}{w_1}}\right) = \begin{cases} 0\,dB & \text{si } w \ll w_1 \\ 3\,dB & \text{si } w = w_1 \\ 20\cdot\log_{10}\left(\frac{w}{w_1}\right) & \text{si } w \gg w_1 \end{cases}\end{cases}$$

cero: $+20$ dB/dec

polo: -20 dB/dec !

Fase (en grados):

$$\operatorname{Arg}(H(jw)) = \operatorname{Arg}(K') + \operatorname{Arg}\left(1+\frac{jw}{z_1}\right) + \operatorname{Arg}\left(1+\frac{jw}{z_2}\right) + \dots$$

$$\operatorname{Arg}\left(1+\frac{jw}{w_1}\right) = \operatorname{arctg}\left(\frac{w}{w_1}\right) = \begin{cases} 0 & , \ w < 0'1\, w_1 \\ 45^\circ & , \ w = w_1 \\ 90^\circ & , \ w > w_1 \end{cases}$$

cero: $+45°/dec$
polo: $-45°/dec$
} afectan 2 décadas: 1 antes y 1 después

Distorsión:

una señal formada por una suma de cosenos de diferentes frecuencias, se dice que no sufre distorsión al atravesar un filtro si todas las componentes (en la banda de paso) sufren la misma atenuación o amplificación y el mismo retardo

si $x(t) = \sum_{m=1}^{M} \cos(w_m t) \Rightarrow y(t) = \sum_{m=1}^{M} |H(jw_m)| \cos(w_m t - \phi_H(w_m))$

distorsión de amplitud: $|H(jw_m)|$

distorsión de fase: retardo τ_m diferente para cada w

$$\tau_m = \frac{\phi_H(w_m)}{w_m} \rightarrow \cos(w_m(t-\tau))$$

si la fase es lineal: todas las componentes sufren el mismo retardo $\Rightarrow$ No hay distorsión de fase

Parámetros de un filtro:

- Bandas de paso: idealmente $|H(jw)| = 1$
- Bandas atenuadas: idealmente $|H(jw)| = 0$
- Bandas de transición: ni fu ni fa

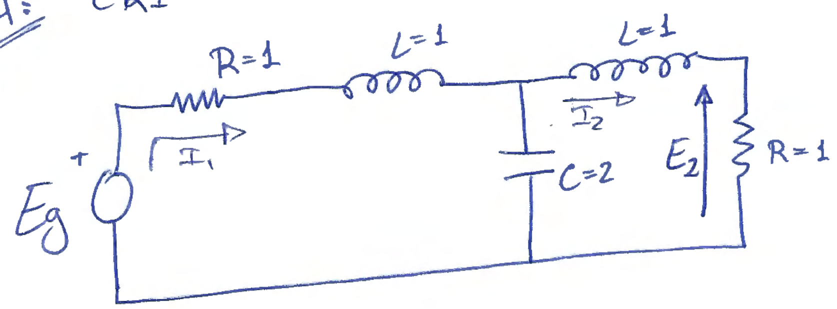
→ <u>Frecuencia de corte:</u> frecuencia donde comienza o acaba una banda del filtro : $-3dB$

→ <u>Rizado:</u> relación entre el máximo y el mínimo de respuesta en la banda de paso o la atenuación mínima en la banda atenuada

Ecualizadores: (de fase)

usados para corregir la distorsión de fase la amplitud es 1 para todas las frecuencias, es un filtro paso todo : $H_{PT}(s)$

Ej 4: CRI



a) $H(s) = \frac{E_2}{E_g}$

b) $h(t)$

c) $H(j\omega)$

d) $e_g(t) = 1 + \cos(10^{-3}t) + \mathrm{sen}(t) + \cos(10t)$

E_2?

a) malla 1: $E_g = (1+s)I_1 + \frac{I_1 - I_2}{2 \cdot s}$; malla 2: $I_2(1+s) + \frac{I_2 - I_1}{2s} = 0$

$E_2 = 1 \cdot I_2$

$$H(s) = \frac{1/2}{(s+1)(s^2+s+1)} = \frac{1}{2s^3+4s^2+4s+2}$$

b) $h(t) = \mathcal{L}^{-1}(H(s))$

$$H(s) = \frac{A}{s+1} + \frac{B}{s-\left(-\frac{1}{2}+j\frac{\sqrt{3}}{2}\right)} + \frac{C}{s-\left(-\frac{1}{2}-j\frac{\sqrt{3}}{2}\right)}$$

$$A = \lim_{s\to -1} H(s)\cdot(s+1) = 1/2$$

$$B = \lim_{s\to -\frac{1}{2}+j\frac{\sqrt{3}}{2}} H(s)\left(s+\frac{1}{2}-j\frac{\sqrt{3}}{2}\right) = \frac{1}{\sqrt{12}}e^{j\pi/6}$$

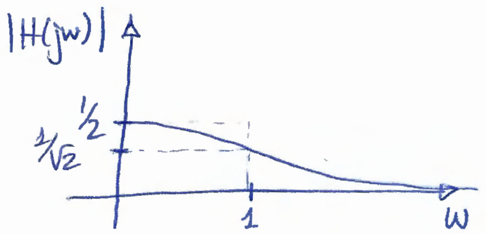
$$C = \qquad = \frac{1}{\sqrt{12}}e^{-j\pi/6}$$

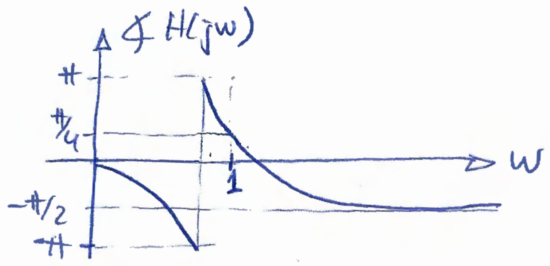
$$h(t) = \frac{1}{2} e^{-t} u(t) + \frac{1}{\sqrt{12}} e^{\left(-\frac{1}{2} + j\frac{\sqrt{3}}{2}\right)t} e^{j\pi/6} u(t) + \frac{1}{\sqrt{12}} e^{\left(-\frac{1}{2} - j\frac{\sqrt{3}}{2}\right)t} e^{-j\pi/6} u(t)$$

$$= \left[\frac{1}{2} e^{-t} + \frac{e^{-t/2}}{\sqrt{3}} \cos\left(\frac{\sqrt{3}}{2}t + \frac{\pi}{6}\right)\right] u(t)$$

c) $$H(j\omega) = \frac{1/2}{-j\omega^3 - 2\omega^2 + 2j\omega + 1} = \frac{1/2}{(1-2\omega^2) + j(2\omega - \omega^3)}$$

$$|H(j\omega)| = \frac{1/2}{\sqrt{(1-2\omega^2)^2 + (2\omega-\omega^3)^2}} = \frac{1/2}{\sqrt{1+\omega^6}}$$





$$\measuredangle H(j\omega) = -tg^{-1}\left(\frac{2\omega - \omega^3}{1 - 2\omega^2}\right)$$

$\omega = 0 \longrightarrow 0$ $\quad \sqrt{2}$

$\omega = 1 \longrightarrow \pi/4$

$\omega = \infty \longrightarrow -\pi/2$

d) $$e_g(t) = \underbrace{1}_{e_{g1}(t)} + \underbrace{\cos(10^{-3}t)}_{e_{g2}(t)} + \underbrace{sen(t)}_{e_{g3}(t)} + \underbrace{\cos(10t)}_{e_{g4}(t)}$$

$$e_{21}(t) = H(j\cdot 0)\cdot 1 = \frac{1}{2}$$

$$E_{g2} = 1 \quad ; \quad E_{22} = H(j\cdot 10^{-3})\cdot 1 = \frac{1/2}{(1+j10^{-3})(1+j\cdot 10^{-3} - 10^{-6})}$$

$$\approx \frac{1/2}{(1+j\cdot 10^{-3})^2} \approx \frac{1}{2}\cdot e^{-j2\cdot 10^{-3}}$$

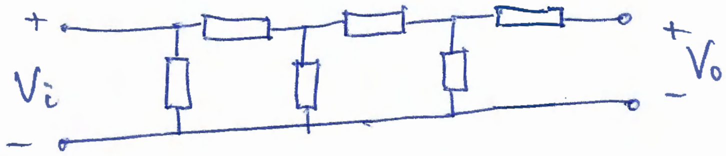
$$e_{22}(t) = \frac{1}{2}\cos(10^{-3}t - 2\cdot 10^{-3}) = \frac{1}{2}\cos(10^{-3}(t-2))$$

$$E_{g3} = -j = e^{-j\pi/2} \quad ; \quad E_{23} = H(j\cdot 1)\, e^{-j\pi/2} = \frac{1}{2\sqrt{2}} e^{-j\pi/4}$$

$$e_{24}(t) = \frac{1}{2} 10^{-3} \cos(10t + j\emptyset)$$

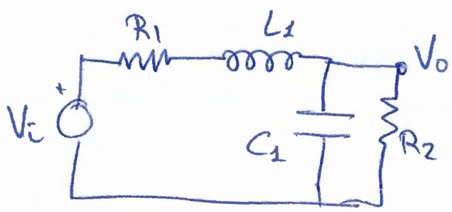
3. Análisis de filtros pasivos

Estructura en escalera:



Para algunos casos de V_i, V_o será nulo. Esto se puede deber a que las ramas serie tengan impedancia infinita o que las ramas paralelo tengan impedancia nula. A estas frecuencias de V_i se les denominan "ceros de transmisión".

Filtro de Butterworth: paso bajo orden 2



$R_1 = 1$

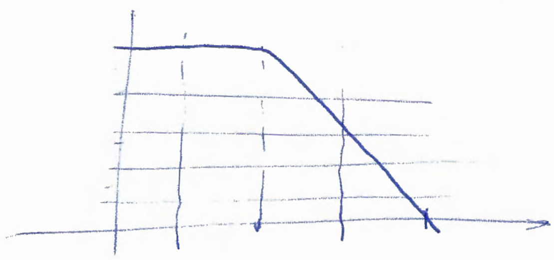
$R_2 = 1$

$L_1 = \sqrt{2}$

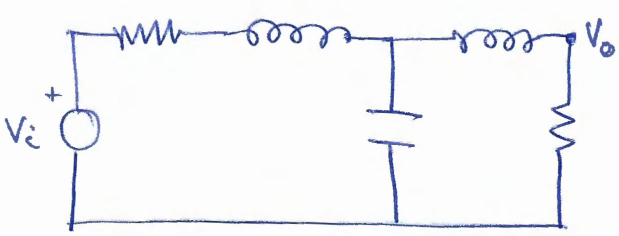
$C_1 = \sqrt{2}$

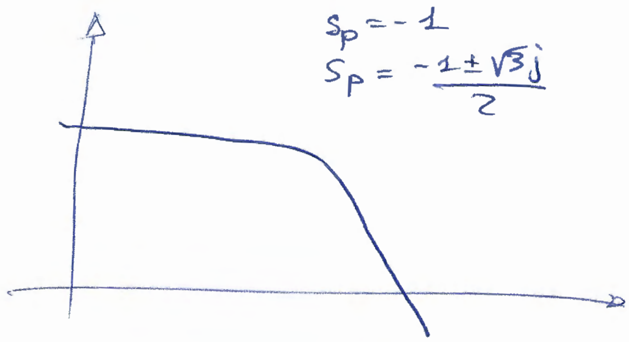
$$\frac{V_o}{V_i} = \frac{1}{2} \cdot \frac{1}{s^2 + \sqrt{2}s + 1}$$

$$S_p = \frac{-1 \pm j}{\sqrt{2}}$$



Filtro de Butterworth: paso bajo orden 3

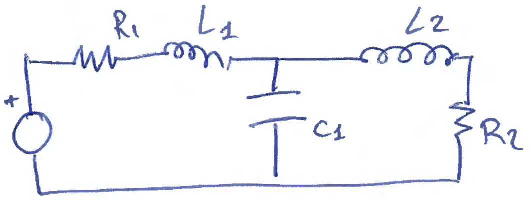


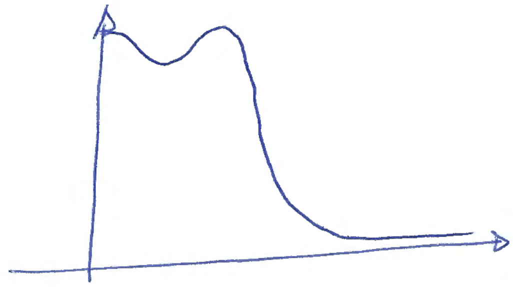


$S_p = -1$

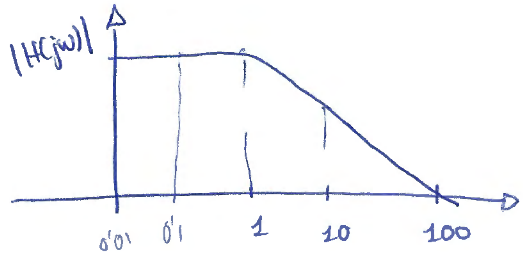
$$S_p = \frac{-1 \pm \sqrt{3}j}{2}$$

Filtro de Chebychev: paso bajo orden 3

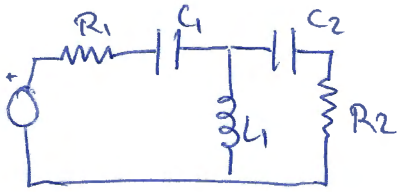




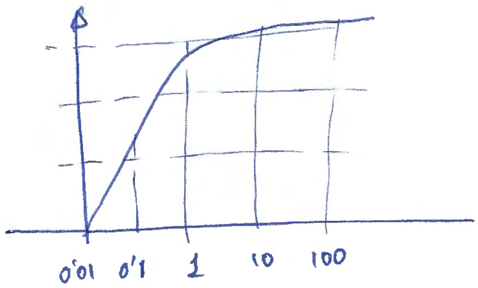
$$H(s) = 2\,\frac{V_o}{V_i} = \frac{1}{(s+\frac{1}{2})(2s^2+s+2)}$$



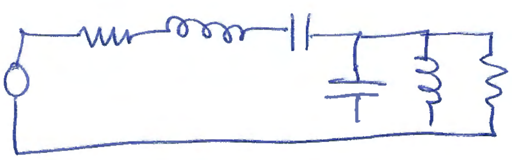
Filtro Butterworth: paso alto orden 3



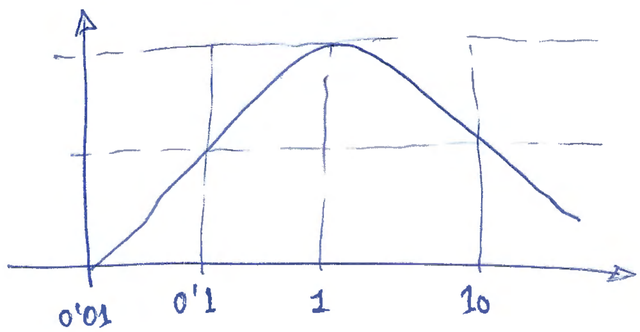
$$H(s) = 2\,\frac{V_o}{V_i} = \frac{s^3}{s^3+2s^2+2s+1} = \frac{s^3}{(s+1)(s^2+s+1)}$$



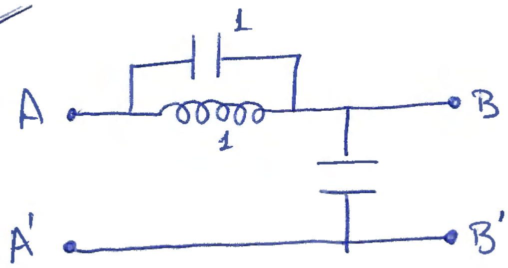
Filtro Butterworth: paso banda orden 2

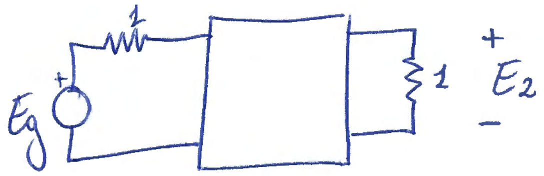


$$H(s) = 2\,\frac{V_o}{V_i} = \frac{s^2}{s^4+\sqrt{2}s^3+3s^2+\sqrt{2}\,s+1}$$



Ej6:





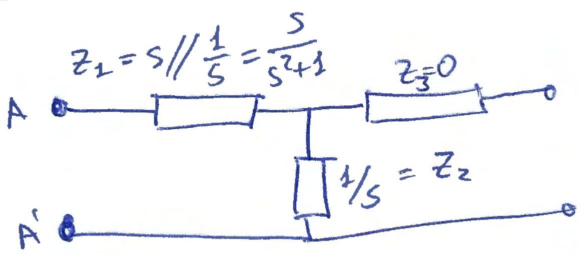
a) [Z]

b) E_2/E_g usando [Z]

c) Razonar tipo de filtro

d) Dibujar $|H(j\omega)| = 2\left|\frac{E_2}{E_g}\right|$

a) hacemos un "equivalente":



$$Z = \begin{bmatrix} Z_1+Z_2 & Z_2 \\ Z_2 & Z_2+Z_3 \end{bmatrix} =$$

$$= \begin{bmatrix} \frac{2s^2+1}{s(s^2+1)} & 1/s \\ 1/s & 1/s \end{bmatrix}$$

b)

$$\frac{E_2}{E_g} = \frac{R_2 \cdot Z_{21}}{(R_g+Z_{11})(R_2+Z_{22}) - Z_{12}Z_{21}} \quad ; \ R_g = R_2 = 1$$

$$\frac{E_2}{E_g} = \frac{s^2+1}{s^3+3s^2+2s+2}$$

c) $Z_1(s) = \frac{s}{s^2+1} \longrightarrow Z_1(j\omega) = \frac{j\omega}{1-\omega^2}$ polo en $\omega = 1$ para Z_1

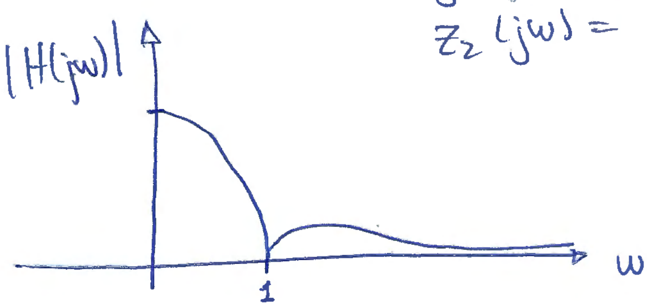
No obstante, para H(s) se "transforma" en 1 cero

ya que si $Z_1(j\omega) = 0 \Rightarrow H(s) = 0$

$Z_2(j\omega) = 1/j\omega \Rightarrow \omega = \infty \Rightarrow$ polo para Z_2

cero para H(s)

d)



4. Análisis de filtros activos

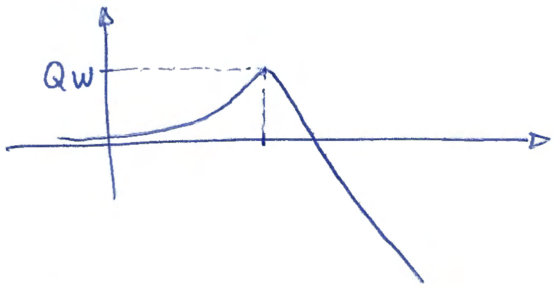
Funciones bicuadráticas:

$$H(s) = \frac{as^2 + bs + c}{\frac{s^2}{w_0^2} + \frac{s}{Qw_0} + 1}$$

w_0 : frecuencia natural
Q : factor de calidad

- Paso bajo: $a = b = 0$, $c = 1$

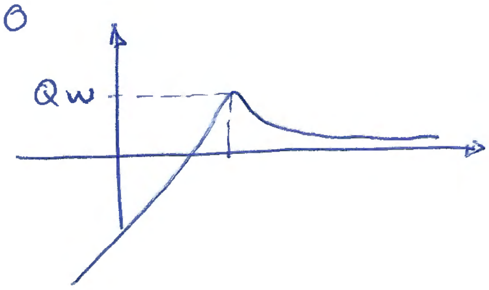
$$H_{PB}(s) = \frac{1}{\frac{s^2}{w_0^2} + \frac{s}{Qw_0} + 1}$$



si : $Q = \frac{1}{\sqrt{2}}$, $w_0 = 1$

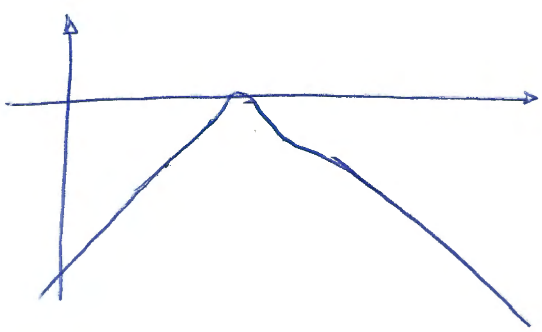
- Paso alto: $a = \frac{1}{w_0^2}$ $b = c = 0$

$$H_{PA}(s) = \frac{s^2/w_0^2}{s^2/w_0^2 + s/Qw_0 + 1}$$

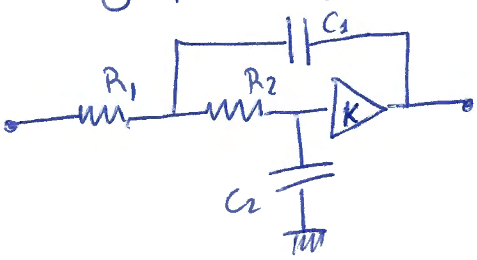


- Paso banda: $a = c = 0$ $b = \frac{1}{w_0 Q}$

$$H_{PB}(s) = \frac{s/Qw_0}{s^2/w_0^2 + s/Qw_0 + 1}$$



Sallen Key paso bajo:

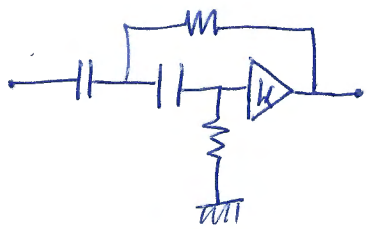


$Z_1 = R_1$
$Z_2 = R_2$
$Z_3 = \frac{1}{sC_1}$
$Z_4 = \frac{1}{sC_2}$

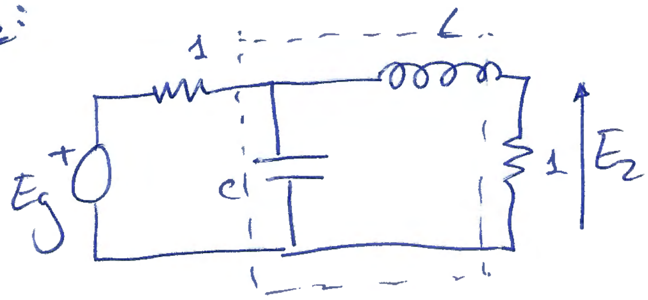
$$H(s) = \frac{1}{\frac{s^2}{W_0^2} + \frac{s}{QW_0} + 1}$$

típicamente $K=1$

Sallen Key paso alto:



Ej 8:



a) $H(s) = 2\frac{E_2}{E_g}$

b) w_0, Q

c) respuesta en amplitud $L=C=\sqrt{2}$; $L=C=\frac{\sqrt{2}}{10}$; $L=\frac{\sqrt{2}}{10}$, $C=10\sqrt{2}$

a) $$Z = \begin{bmatrix} 1/Cs & 1/Cs \\ 1/Cs & Ls+\frac{1}{Cs} \end{bmatrix} = \begin{bmatrix} 1/C\cdot s & 1/C\cdot s \\ 1/C\cdot s & \frac{LCs^2+1}{C\cdot s} \end{bmatrix}$$

$$H(s) = 2 \cdot \frac{R \cdot s \cdot Z_{21}}{(R_g + Z_{11})(R_s + Z_{22}) - Z_{12} Z_{21}} = \frac{1}{\frac{LC}{2} s^2 + \frac{L+C}{2} \cdot s + 1}$$

$$= \frac{1}{s^2/\omega_0^2 + s/Q\omega_0 + 1}$$

b)

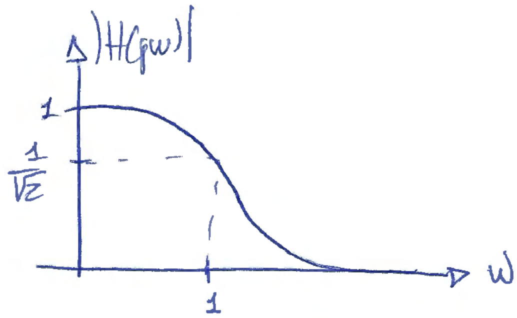
$$\omega_0^2 = \frac{2}{LC} \rightarrow \omega_0 = \sqrt{\frac{2}{LC}}$$

$$Q\omega_0 = \frac{2}{L+C} \longrightarrow Q = \frac{2}{(L+C)\sqrt{\frac{2}{LC}}} = \frac{\sqrt{2}}{\sqrt{\frac{L}{C}} + \sqrt{\frac{C}{L}}}$$

c) $L = C = \sqrt{2}$

$$\omega_0 = \frac{2}{\sqrt{2}\sqrt{2}} = 1 \qquad H(s) = \frac{1}{s^2 + \sqrt{2} \cdot s + 1}$$

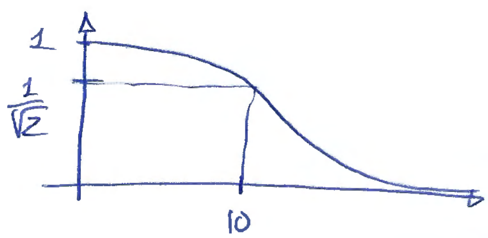
$$Q = \frac{\sqrt{2}}{\sqrt{\frac{\sqrt{2}}{\sqrt{2}}} + \sqrt{\frac{\sqrt{2}}{\sqrt{2}}}} = \frac{1}{\sqrt{2}}$$



$L = C = \sqrt{2}/10$

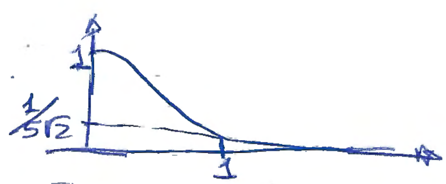
$$\omega_0 = \sqrt{\frac{2}{\sqrt{2}/10 \cdot \sqrt{2}/10}} = 10$$

$$Q = \frac{\sqrt{2}}{1+1} = \frac{1}{\sqrt{2}}$$



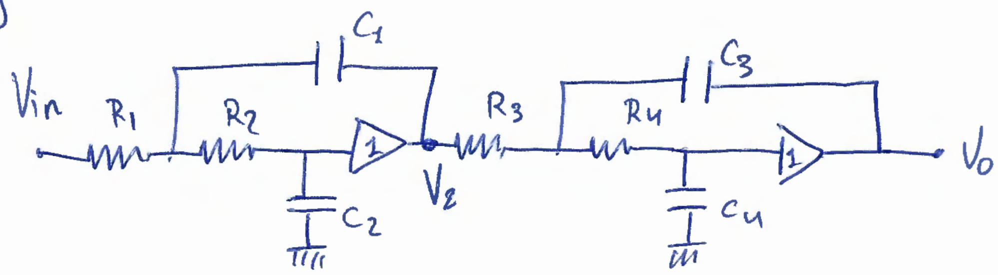
$L = \sqrt{2}/10 \quad , \quad C = 10\sqrt{2}$

$$\omega_0 = \sqrt{\frac{2}{\sqrt{2}/10 \cdot 10\sqrt{2}}} = 1$$



$$Q = \frac{\sqrt{2}}{\sqrt{\frac{\sqrt{2}/10}{10\sqrt{2}}} + \sqrt{\frac{10 \cdot \sqrt{2}}{\sqrt{2}/10}}} \simeq \frac{\sqrt{2}}{\sqrt{\frac{10\sqrt{2}}{\sqrt{2}/10}}} = \frac{\sqrt{2}}{10} = \frac{1}{5\sqrt{2}}$$

Ej



a) $H(s) = \dfrac{V_o(s)}{V_{in}(s)}$

$$\omega_{o1} = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}}$$

b) $\omega_{o1} = \omega_{o2} = 1$

$Q_1 = Q_2 = 1/\sqrt{2}$

¿$|H(j\omega)|$?

$$Q_1 = \frac{\sqrt{R_1 R_2 C_1}}{(R_1+R_2)\sqrt{C_2}}$$

$$\omega_{o2} = \frac{1}{\sqrt{R_3 R_4 C_3 C_4}}$$

c) $\omega_{o1} = \omega_{o2} = 1$

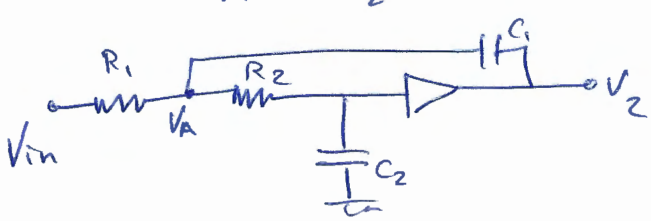
$$Q_1 = \frac{1}{-2\cos\left(\frac{5\pi}{8}\right)}$$

$$Q_2 = \frac{\sqrt{R_3 R_4 C_3}}{(R_3+R_4)\sqrt{C_4}}$$

$$Q_2 = \frac{1}{-2\cos\left(\frac{7\pi}{8}\right)}$$

¿$|H(j\omega)|$?

a) $H(s) = \dfrac{V_2}{V_{in}} \cdot \dfrac{V_o}{V_2} = \dfrac{V_o}{V_{in}}$



$$\begin{cases} \dfrac{V_{in} - V_A}{R_1} + (V_2 - V_A)(C_1 \cdot s) + \dfrac{V_2 - V_A}{R_2} = 0 \\ \dfrac{V_A - V_2}{R_2} + (-V_2)\cdot(C_2 \cdot s) = 0 \end{cases}$$

$$\frac{V_2}{V_{in}} = \frac{1}{R_1 R_2 C_1 C_2 s^2 + (R_1+R_2) C_2 \cdot s + 1} =$$

$$\frac{V_2}{V_{in}} = \frac{1}{\frac{s^2}{W_{01}^2} + \frac{s}{Q_1 W_{01}} + 1}$$

La célula 2ª es "igual" que la 1ª, luego:

$$\frac{V_o}{V_2} = \frac{1}{\frac{s^2}{W_{02}^2} + \frac{s}{Q_2 W_{02}} + 1}$$

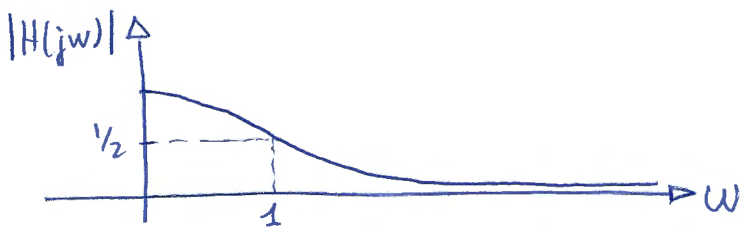
$$H(s) = \frac{V_o}{V_{in}} = \frac{V_o}{V_2} \cdot \frac{V_2}{V_{in}} = \frac{1}{\left(s^2/W_{01}^2 + \frac{s}{Q_1 W_{01}} + 1\right) \cdot \left(\frac{s^2}{W_{02}^2} + \frac{s}{Q_2 W_{02}} + 1\right)}$$

b) $W_{01} = W_{02} = 1$; $Q_1 = Q_2 = 1/\sqrt{2}$

$$H(s) = \underbrace{\frac{1}{s^2 + \sqrt{2}s + 1}} \cdot \frac{1}{s^2 + \sqrt{2}s + 1}$$

$$\hookrightarrow \frac{1}{1 - W^2 + \sqrt{2}jW} \Rightarrow |\cdot| \Rightarrow \frac{1}{\sqrt{(1-W^2)^2 + 2W^2}} = \frac{1}{\sqrt{1 + W^4}}$$

$$|H(jW)| = \left(\frac{1}{\sqrt{1+W^4}}\right)^2 = \frac{1}{1+W^4}$$



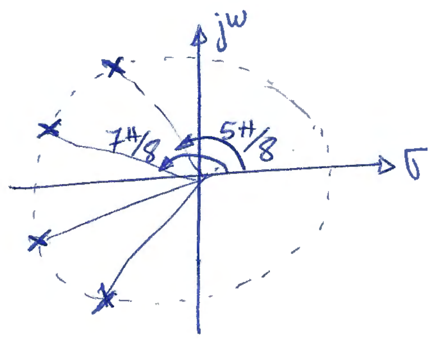
c) $W_{01} = 1 = W_{02}$; $Q_1 = \frac{1}{-2\cos\left(\frac{5\pi}{6}\right)}$ $Q_2 = \frac{1}{-2\cos\left(\frac{7\pi}{6}\right)}$

$$H(s) = \underbrace{\frac{1}{s^2 - 2\cos\left(\frac{5\pi}{8}\right)s + 1}}_{H_1(s)} \cdot \underbrace{\frac{1}{s^2 - 2\cos\left(\frac{7\pi}{8}\right) \cdot s + 1}}_{H_2(s)}$$

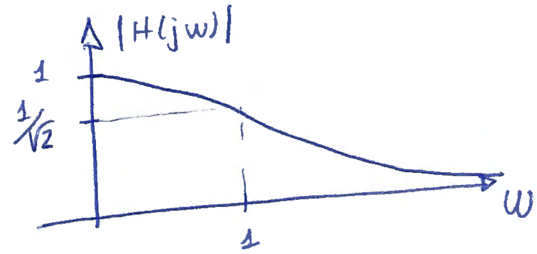
$$H'(s) = \frac{1}{s^2 - 2\cos(\theta)\cdot s + 1}$$

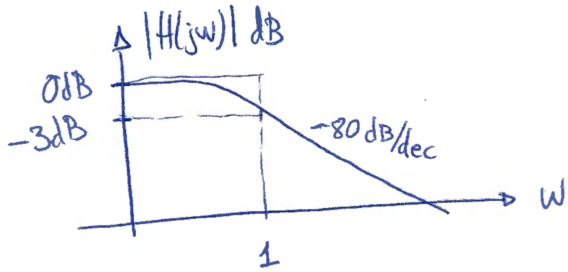
$$\hookrightarrow s = \frac{\cos(\theta) \pm \sqrt{4\cos^2\theta - 4}}{2} = \frac{\cos(\theta) \pm \sqrt{-4\,\mathrm{sen}^2\theta}}{2} =$$

$$= \cos(\theta) \pm j\,\mathrm{sen}(\theta) = e^{\pm j\theta} \;; \quad \theta = \begin{cases} \to 5\pi/8 \\ \to 7\pi/8 \end{cases}$$



$$|H(jw)| = \frac{1}{\sqrt{1+w^8}}$$





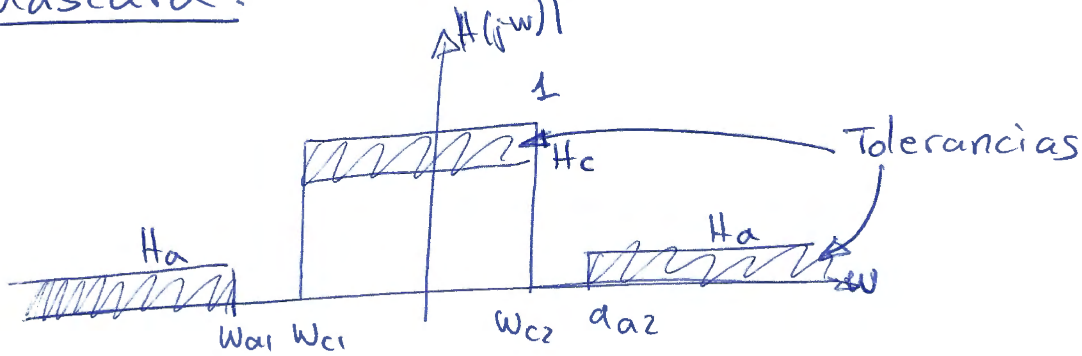
Tema 3: Diseño de filtros

1. Características de los filtros

$$|H(jw)|^2 = H(jw) \cdot H^*(jw) = H(jw) \cdot H(-jw) = H(s) \cdot H(-s)$$

a partir del módulo en amplitud tomamos los polos del semiplano Izquierdo y la mitad de los ceros del eje jw (por pares conjugados)

Máscara:



$H_c \leq |H(jw)| \leq 1$ para las bandas de paso

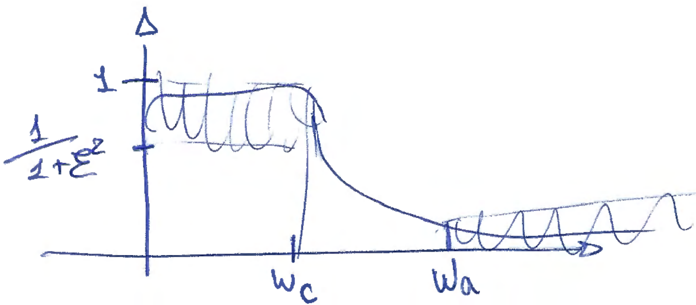
$H_a \leq |H(jw)| \leq H_c$ para las bandas de transición

$0 \leq |H(jw)| \leq H_a$ para las bandas atenuadas

Aproximación paso bajo:

$$|H(jw)|^2 = \frac{1}{1 + \varepsilon^2 \cdot F_n\left(\frac{w}{w_c}\right)} \quad ; \quad 0 \leq F_n^2(x) \leq 1 \text{ para } 0 \leq x \leq 1$$

$$F_n^2(x) \gg 1 \quad , \quad x > 1$$



Parámetros:

$\rightarrow 10 \log(1+\varepsilon^2) = \alpha_c$

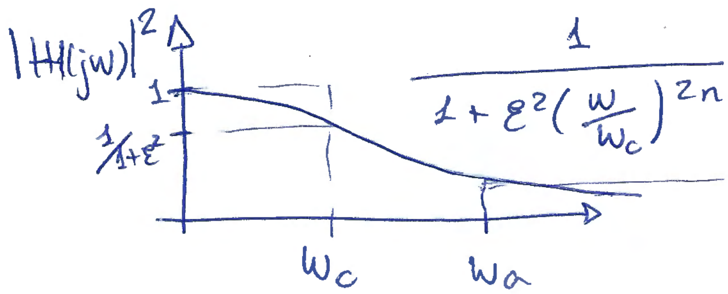
$\rightarrow 10 \log\left(1+\varepsilon^2 F_n^2(w_a/w_c)\right) = \alpha_a$

$$\varepsilon = \sqrt{10^{\alpha_c/10} - 1}$$

$$F_n(w_a/w_c) = \sqrt{\frac{10^{\alpha_a/10} - 1}{10^{\alpha_c/10} - 1}} = \frac{\sqrt{10^{\alpha_a/10} - 1}}{\varepsilon}$$

Para hallar n debemos conocer $F_n(x)$

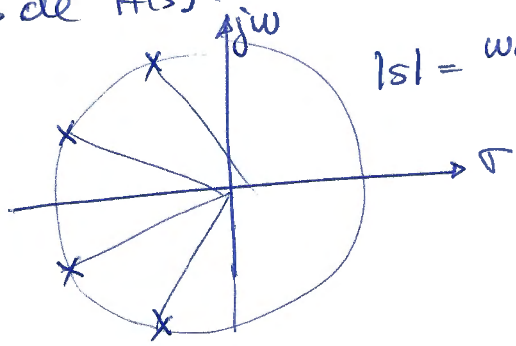
Butterworth: $F_n(x) = x^n$



$$\frac{1}{1+\varepsilon^2\left(\frac{w}{w_c}\right)^{2n}} = |H(jw)|^2$$

$$|H(s).H(-s)| = \frac{1}{1+\varepsilon^2\left(\frac{-s^2}{w_c^2}\right)^n}$$

polos de H(s):



$$|s| = w_c/\sqrt[n]{\varepsilon_c}$$

Chebycheff:

$$F_n(x) = \begin{cases} \cos(n \cdot \arccos(x)) & 0 \leq x \leq 1 \\ \cosh(n \cdot \text{arccosh}(x)) & x > 1 \end{cases}$$

orden 0 $\Rightarrow$ $F_0(x) = 1$

orden 1 $\Rightarrow$ $F_1(x) = x$

orden 2 $\Rightarrow$ $F_2(x) = 2x^2 - 1$

$$F_{n+1}(x) = 2x \cdot F_n(x) - F_{n-1}(x)$$

$$|H(j\omega)|^2 = \frac{1}{1 + \varepsilon^2 \cdot F_n^2\left(\frac{\omega}{\omega_c}\right)}$$